Re-Exam Kaleidoscope Modern Physics

28 november 2013, 18:30-21:30, Aletta Jacobshal

ONLY OPEN THE EXAM WHEN INSTRUCTED BY THE SUPERVISOR

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- Answer all questions short and to the point, but complete; write legible.
- Use of a calculator is not allowed.
- $hc = 1240$ eV·nm ; $\hbar c = 200$ eV·nm ; 1 u = 931.5 MeV/c^2.
- Put your name and student number on each sheet. Good luck!

1. How can the spectrum of hydrogen contain so many lines when hydrogen contains only one electron? **(1 point)**
   The single electron in hydrogen can occupy many quantum states, each of which has a different energy. If an electron changes it's state, a photon is emitted as a result. The energy and hence the wavelength of the emitted photon is determined by the energy difference of the initial and final states of the electron. An ensemble of hydrogen atoms may all occupy different quantum states and undergo transitions, hence a complete specturm is observed.

2. The average kinetic energy of an atom or molecule is about $K = \frac{3}{2}kT$ with k Boltzmann's constant. At what temperature will K be on the order of the bond energy (and hence the bond easily broken by thermal motion) for *(a)* a covalent bond (say H_2) of a binding energy 4.0 eV, and *(b)* a "weak" hydrogen bond of binding energy 0.12 eV? **(2 points)**
   The requirement for breaking the bond is that the kinetic energy equals the binding energy, $K = \frac{3}{2}kT = E_{bind}$. Thus $T = \frac{2}{3}E_{bind}/k$. From the appendix $k = 1.38 \cdot 10^{-23}\, J/K = 1.38 \cdot 10^{-23}\, J/K \times 6.24 \cdot 10^{18}\, eV/J = 8.5 \cdot 10^{-5}\, eV/K$. So, $\frac{2}{3}/k = 7800 K/eV$. Breaking the covalent bond thus requires a temperature of $T = 7800 \times 4 = 31,000 K$. Breaking the hydrogen bond requires a temperature of $T = 940\, K$.

3. The decay of the isotope ^{20}Na into ^{20}Ne proceeds in two steps. In the second step only a photon with $E_\gamma = 1.634$ MeV is emitted. In the first step a β-particle is emitted. What is the charge of the β particle and which other particles are emitted (if any)? The nuclear masses of ^{20}Na and ^{20}Ne are 20.007351 u and 19.9924401754 u, respectively. What is the (maximum) energy of the β-particle in MeV? *Hint*: you may neglect the motion of the daughter nucleus; $\vec{p} = m\vec{v}$. **(3 points)**
   I. In gamma-decay only a photon is emitted and the number of protons and neutrons does not change. Hence the intermediate nucleus is an excited state of Ne. From the appendix, we find that for Na $Z = 11$, whereas for Ne $Z = 10$. Hence once unit of charge is lost from the nucleus, which must be taken by the beta, which is thus a e^+. **II. In beta-decay also a neutrino must be produced to conserve lepton number. III. The energy release going from ^{20}Na to ^{20}Ne is $Q = (20.007351 - 19.9924401754 - 0.000549)\, u = 1.5 \cdot 10^{-2}\, u = 13.3\, MeV$. Note that also the positron must be created and hence it's mass! If we ignore the motion of the daughter nucleus, this energy is split between the neutrino and positron. Momentum conservation requires that the electron and neutrino have equal but opposite momenta. Because Q is a lot larger than the electron mass, we may neglect the latter. Hence the maximum energy of the electron is $E_{e,max} = 13.3/2 = 6.65$ MeV.**

4. Briefly explain the principle of ^{14}C-dating. Can it be used for dating: *(i)* wine, *(ii)* rock walls of an old civilization, *(iii)* wall paintings of *homo sapiens*, *(iv)* the bones of a Neanderthaler? Why (not)? **(2 points)**
   I. Carbon-14 is continuously produced in the upper atmosphere by cosmic radiation. It is absorbed

by living organisms and incorported into their system. Once they die, the intake of C-14 stops and the accumulated amount slowly drops because of radioactive decay. C-14 has a halflife of about 6000 years. **II.** *i* wine: consists of grapes, which accumulate C-14 while they are on the plant. Wine is kept in closed bottles, thus no new carbon is accumulated. Hence it can be carbon-dated. *ii* rock walls: do not "grow" and hence do not accumulate C-14 during their "life". Hence: no. *iii* wall paintings: if the used paint is organic (plants, ashes), then yes, otherwise: no. *iv* bones can be radio-dated, because they contain carbon, so: yes.

5. Within the context of the free-electron model, explain what a conductor, isolator and semi-conductor are. Sketch the band-structure. Give an example of each. **(2 points)**
In the free-electron model, the electrons can be found in bands, which may be full, partly full or empty. If the band is only partly filled, we have a conductor. For an isolator, the band is completely full and the next higher band is separated from this band by a large gap. If this band is this gap is very small, we have a semi-conductor. Pictures as Fig. 40-32 on pg. 1091 of the book.

6. Does the presence of a neutrino among the decay products of a particle necessarily mean that the decay occurs via the weak interaction? Do all decays via the weak interaction produce a neutrino? Explain. **(1 point)**
Yes, neutrinos only interact via the weak interaction. Not all weak decays necessarily produce neutrinos, *e.g.* $\Lambda \to p + \pi^-$.

7. Sketch the energy levels of a Li^{2+} ion ($Z = 3$). What is the ionisation energy in eV? Calculate the wavelength and frequency of a photon that can ionize a Li^{2+} ion in the groundstate and that gives the ejected electron a kinetic energy of 5 eV. **(3 points)**
I. This ion is hydrogen-like, hence the energy level distribution looks similar. However, Z is larger and the energy levels scale as Z^2. The ionization energy is thus 13.6 eV times 9 = 122 eV. **II.** The corresponding energy levels are $E_n = -122/n^2$. **III.** The photon must have an energy of $122 + 5 = 127$ eV. For photons is holds that $\lambda = \frac{hc}{E} = \frac{1240\,eV \cdot nm}{E}$. So the wavelength is $\lambda = 1240/127 \simeq 10\,nm$.

8. The Z^0 boson, discovered in 1985, is the mediator of the weak nuclear force, and it typically decays very quickly. Its average rest energy is 91 GeV, but its short lifetime shows up as an intrinsic width of 2.5 GeV. What is the lfietime of this particle? **(2 points)**
The intrinsic width is determined from the lifetime of the Z boson. For energy and time, the uncertainty relation is $\Delta E \Delta t \geq \hbar/2$. Thus the lifetime of the Z-boson is $\tau = \hbar/2E$. We know $\hbar \cdot c = 200 MeV \cdot fm$, thus with $c = 3 \cdot 10^8$ m/s, $\hbar \simeq 6.7 \cdot 10^{-22}$ MeVs. The lifetime then becomes $\tau = 6.7 \cdot 10^{-22}/5 \cdot 10^3 = 1.35 \cdot 10^{-25}$ s.

9. The maximum number of electrons k contained in an atomic sub-shell with principle quantum number n and angular momentum l is $k = 2(2l+1)$. Explain the first factor 2 and the factor $(2l+1)$. For a given n, which values of l are allowed? Which of the following electron configurations are <u>not</u> allowed: (a) $1s^2 2s^2 2p^4 3s^2 4p^2$; (b) $1s^2 2s^2 2p^8 3s^1$; (c) $1s^2 2s^2 2p^6 3s^2 3p^5 4s^2 4d^5 4f^1$. If not allowed, explain why. **(2 points)**
The first factor 2 accounts for the two spin states per electron: spin-up or spin-down. The second factor $2l + 1$ accounts for the various orbital angular momentum states, $|m| \leq l$. For a given n, $l < n$. The states s, p, d, f have $l = 0, 1, 2, 3$, respectively, with $n \leq 2, 6, 10, 14$. Thus states (a) and (c) are allowed. State (b) claims to have a $2p^8$, which is not allowed.

10. Fermi-problem: Does the amount of rain collected on the rooftops in the Netherlands suffice to provide everyone in the Netherlands with drinking water? **(3 points)**

Approach: estimate amount of rain fall per area in the NL; the combined area of the rooftops; the number of NL people; the amount of drinkwater consumed by a NL peson. Alternatively, estimate the average amount of rooftop per person. Annual rainfall in the NL is about 700 mm per year. This can be estimated from the number of days that it rains (1 out of 2 to 3) and the average amount of rain per rainy day (no more than 1 cm). The average amount of rooftop per person is about 15 m^2 from their house. We also have to take into account schools, shops, work, etc. Let's take a factor 3 (more than 1, less than 10) to arrive at a total area of about 50 m^2 per person. At 0.7 m/year per year this comes to a total 35 m^3/year, or about 100 l per day. Average amount of water used for drinking is definitely less than that, of order of several l per person per day. If "drinking water" is interpreted as total water consumption: that is about 130 l per person per day. This can be estimated from drinking, cooking, showering, toilet use, dishwashing, etc.

Fundamental Constants

Quantity	Symbol	Approximate Value	Current Best Value†
Speed of light in vacuum	c	3.00×10^8 m/s	2.99792458×10^8 m/s
Gravitational constant	G	6.67×10^{-11} N·m²/kg²	$6.6728(67) \times 10^{-11}$ N·m²/kg²
Avogadro's number	N_A	6.02×10^{23} mol^{-1}	$6.02214179(30) \times 10^{23}$ mol^{-1}
Gas constant	R	8.314 J/mol·K = 1.99 cal/mol·K = 0.0821 L·atm/mol·K	8.314472(15) J/mol·K
Boltzmann's constant	k	1.38×10^{-23} J/K	$1.3806504(24) \times 10^{-23}$ J/K
Charge on electron	e	1.60×10^{-19} C	$1.602176487(40) \times 10^{-19}$ C
Stefan-Boltzmann constant	σ	5.67×10^{-8} W/m²·K⁴	$5.670400(40) \times 10^{-8}$ W/m²·K⁴
Permittivity of free space	$\epsilon_0 = (1/c^2\mu_0)$	8.85×10^{-12} C²/N·m²	$8.854187817\ldots \times 10^{-12}$ C²/N·m²
Permeability of free space	μ_0	$4\pi \times 10^{-7}$ T·m/A	$1.2566370614\ldots \times 10^{-6}$ T·m/A
Planck's constant	h	6.63×10^{-34} J·s	$6.62606896(33) \times 10^{-34}$ J·s
Electron rest mass	m_e	9.11×10^{-31} kg = 0.000549 u = 0.511 MeV/c^2	$9.10938215(45) \times 10^{-31}$ kg = $5.4857990943(23) \times 10^{-4}$ u
Proton rest mass	m_p	1.6726×10^{-27} kg = 1.00728 u = 938.27 MeV/c^2	$1.672621637(83) \times 10^{-27}$ kg = 1.00727646677(10) u
Neutron rest mass	m_n	1.6749×10^{-27} kg = 1.008665 u = 939.57 MeV/c^2	$1.674927211(84) \times 10^{-27}$ kg = 1.00866491597(43) u
Atomic mass unit (1 u)		1.6605×10^{-27} kg = 931.49 MeV/c^2	$1.660538782(83) \times 10^{-27}$ kg = 931.494028(23) MeV/c^2

† CODATA (3/07), Peter J. Mohr and Barry N. Taylor, National Institute of Standards and Technology. Numbers in parentheses indicate one-standard-deviation experimental uncertainties in final digits. Values without parentheses are exact (i.e., defined quantities).

Other Useful Data

Joule equivalent (1 cal)	4.186 J
Absolute zero (0 K)	−273.15°C
Acceleration due to gravity at Earth's surface (avg.)	9.80 m/s² (= g)
Speed of sound in air (20°C)	343 m/s
Density of air (dry)	1.29 kg/m³
Earth: Mass	5.98×10^{24} kg
Radius (mean)	6.38×10^3 km
Moon: Mass	7.35×10^{22} kg
Radius (mean)	1.74×10^3 km
Sun: Mass	1.99×10^{30} kg
Radius (mean)	6.96×10^5 km
Earth–Sun distance (mean)	149.6×10^6 km
Earth–Moon distance (mean)	384×10^3 km

The Greek Alphabet

Alpha	Α	α	Nu	Ν	ν
Beta	Β	β	Xi	Ξ	ξ
Gamma	Γ	γ	Omicron	Ο	o
Delta	Δ	δ	Pi	Π	π
Epsilon	Ε	ϵ, ε	Rho	Ρ	ρ
Zeta	Ζ	ζ	Sigma	Σ	σ
Eta	Η	η	Tau	Τ	τ
Theta	Θ	θ	Upsilon	Υ	υ
Iota	Ι	ι	Phi	Φ	ϕ, φ
Kappa	Κ	κ	Chi	Χ	χ
Lambda	Λ	λ	Psi	Ψ	ψ
Mu	Μ	μ	Omega	Ω	ω

Values of Some Numbers

$\pi = 3.1415927$	$\sqrt{2} = 1.4142136$	$\ln 2 = 0.6931472$	$\log_{10} e = 0.4342945$
$e = 2.7182818$	$\sqrt{3} = 1.7320508$	$\ln 10 = 2.3025851$	1 rad = 57.2957795°

Mathematical Signs and Symbols

$\propto$	is proportional to	$\leq$	is less than or equal to
$=$	is equal to	$\geq$	is greater than or equal to
$\approx$	is approximately equal to	Σ	sum of
$\neq$	is not equal to	$\bar{x}$	average value of x
$>$	is greater than	Δx	change in x
$\gg$	is much greater than	$\Delta x \to 0$	Δx approaches zero
$<$	is less than	$n!$	$n(n-1)(n-2)\ldots(1)$
$\ll$	is much less than		

Properties of Water

Density (4°C)	1.000×10^3 kg/m³
Heat of fusion (0°C)	333 kJ/kg (80 kcal/kg)
Heat of vaporization (100°C)	2260 kJ/kg (539 kcal/kg)
Specific heat (15°C)	4186 J/kg·C° (1.00 kcal/kg·C°)
Index of refraction	1.33

Periodic Table of the Elements§

Group I	Group II	Transition Elements										Group III	Group IV	Group V	Group VI	Group VII	Group VIII
H 1 1.00794 $1s^1$																	He 2 4.002602 $1s^2$
Li 3 6.941 $2s^1$	Be 4 9.012182 $2s^2$											B 5 10.811 $2p^1$	C 6 12.0107 $2p^2$	N 7 14.0067 $2p^3$	O 8 15.9994 $2p^4$	F 9 18.9984032 $2p^5$	Ne 10 20.1797 $2p^6$
Na 11 22.98976928 $3s^1$	Mg 12 24.3050 $3s^2$											Al 13 26.9815386 $3p^1$	Si 14 28.0855 $3p^2$	P 15 30.973762 $3p^3$	S 16 32.065 $3p^4$	Cl 17 35.453 $3p^5$	Ar 18 39.948 $3p^6$
K 19 39.0983 $4s^1$	Ca 20 40.078 $4s^2$	Sc 21 44.955912 $3d^14s^2$	Ti 22 47.867 $3d^24s^2$	V 23 50.9415 $3d^34s^2$	Cr 24 51.9961 $3d^54s^1$	Mn 25 54.938045 $3d^54s^2$	Fe 26 55.845 $3d^64s^2$	Co 27 58.933195 $3d^74s^2$	Ni 28 58.6934 $3d^84s^2$	Cu 29 63.546 $3d^{10}4s^1$	Zn 30 65.409 $3d^{10}4s^2$	Ga 31 69.723 $4p^1$	Ge 32 72.64 $4p^2$	As 33 74.92160 $4p^3$	Se 34 78.96 $4p^4$	Br 35 79.904 $4p^5$	Kr 36 83.798 $4p^6$
Rb 37 85.4678 $5s^1$	Sr 38 87.62 $5s^2$	Y 39 88.90585 $4d^15s^2$	Zr 40 91.224 $4d^25s^2$	Nb 41 92.90638 $4d^45s^1$	Mo 42 95.94 $4d^55s^1$	Tc 43 (98) $4d^55s^2$	Ru 44 101.07 $4d^75s^1$	Rh 45 102.90550 $4d^85s^1$	Pd 46 106.42 $4d^{10}5s^0$	Ag 47 107.8682 $4d^{10}5s^1$	Cd 48 112.411 $4d^{10}5s^2$	In 49 114.818 $5p^1$	Sn 50 118.710 $5p^2$	Sb 51 121.760 $5p^3$	Te 52 127.60 $5p^4$	I 53 126.90447 $5p^5$	Xe 54 131.293 $5p^6$
Cs 55 132.9054519 $6s^1$	Ba 56 137.327 $6s^2$	57–71†	Hf 72 178.49 $5d^26s^2$	Ta 73 180.94788 $5d^36s^2$	W 74 183.84 $5d^46s^2$	Re 75 186.207 $5d^56s^2$	Os 76 190.23 $5d^66s^2$	Ir 77 192.217 $5d^76s^2$	Pt 78 195.084 $5d^96s^1$	Au 79 196.966569 $5d^{10}6s^1$	Hg 80 200.59 $5d^{10}6s^2$	Tl 81 204.3833 $6p^1$	Pb 82 207.2 $6p^2$	Bi 83 208.98040 $6p^3$	Po 84 (209) $6p^4$	At 85 (210) $6p^5$	Rn 86 (222) $6p^6$
Fr 87 (223) $7s^1$	Ra 88 (226) $7s^2$	89–103‡	Rf 104 (267) $6d^27s^2$	Db 105 (268) $6d^37s^2$	Sg 106 (271) $6d^47s^2$	Bh 107 (272) $6d^57s^2$	Hs 108 (277) $6d^67s^2$	Mt 109 (276) $6d^77s^2$	Ds 110 (281) $6d^97s^1$	Rg 111 (280) $6d^{10}7s^1$	112 (285) $6d^{10}7s^2$						

Symbol — Cl 17 — Atomic Number
Atomic Mass§ — 35.453
$3p^5$ — Electron Configuration (outer shells only)

†Lanthanide Series	La 57 138.90547 $5d^16s^2$	Ce 58 140.116 $4f^15d^16s^2$	Pr 59 140.90765 $4f^35d^06s^2$	Nd 60 144.242 $4f^45d^06s^2$	Pm 61 (145) $4f^55d^06s^2$	Sm 62 150.36 $4f^65d^06s^2$	Eu 63 151.964 $4f^75d^06s^2$	Gd 64 157.25 $4f^75d^16s^2$	Tb 65 158.92535 $4f^95d^06s^2$	Dy 66 162.500 $4f^{10}5d^06s^2$	Ho 67 164.93032 $4f^{11}5d^06s^2$	Er 68 167.259 $4f^{12}5d^06s^2$	Tm 69 168.93421 $4f^{13}5d^06s^2$	Yb 70 173.04 $4f^{14}5d^06s^2$	Lu 71 174.967 $4f^{14}5d^16s^2$
‡Actinide Series	Ac 89 (227) $6d^17s^2$	Th 90 232.03806 $6d^27s^2$	Pa 91 231.03588 $5f^26d^17s^2$	U 92 238.0289 $5f^36d^17s^2$	Np 93 (237) $5f^46d^17s^2$	Pu 94 (244) $5f^66d^07s^2$	Am 95 (243) $5f^76d^07s^2$	Cm 96 (247) $5f^76d^17s^2$	Bk 97 (247) $5f^96d^07s^2$	Cf 98 (251) $5f^{10}6d^07s^2$	Es 99 (252) $5f^{11}6d^07s^2$	Fm 100 (257) $5f^{12}6d^07s^2$	Md 101 (258) $5f^{13}6d^07s^2$	No 102 (259) $5f^{14}6d^07s^2$	Lr 103 (262) $5f^{14}6d^17s^2$

§ Atomic mass values averaged over isotopes in percentages they occur on Earth's surface. For many unstable elements, mass of the longest-lived known isotope is given in parentheses. 2006 revisions. (See also Appendix F.) Preliminary evidence (unconfirmed) has been reported for elements 113, 114, 115, 116 and 118.